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EFFECT OF LEADING EDGE ON FREE-CONVECTION HEAT TRANSFER

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The leading edge affects heat transfer at moderate values of the Grashof number. The results of calculations are compared with experimental results.

The experimental investigation of free-convection heat transfer shows that the experimental data for the heat-transfer coefficient deviate significantly and systematically from the curve determined by boundary-layer theory in the region $Gr < 10^4$ [1]. This is consistent with general ideas on the nature of free-convection heat transfer, according to which there is an induced flow upstream of the leading edge, which causes intensification of heat transfer [2, 3]. It was shown in [4] by integral methods that in the case of an isothermal plate the leading edge affects the relative vertical position of the boundary layer on the plate, but not the velocity and temperature profiles.

The effect of the leading edge is manifested at moderate values of the Grashof number, when the interaction of the boundary layer with the external flow must be taken into account. This interaction is obtained by the method of matched asymptotic expansions, and for the local heat-transfer coefficient at Pr = 0.72 we have [5]

$$\mathrm{Nu}_x = 0.356\,\mathrm{Gr}_x^{1/4} + c_1\,0.072\,\mathrm{Gr}_x^{-1/12} + 0.891\,\mathrm{Gr}_x^{-1/4} + O(\mathrm{Gr}_x^{-1/2}), \tag{1}$$

where c_1 is an indeterminate constant in the characteristic solution. The obtained solution has a singularity on the leading edge and represents the flow only in the region $x > 0$. To shift the singularities in the direction of their true position, we deform the longitudinal coordinate x by the formula

$$x = X + \mathrm{Gr}^{-1/4} f(X,\ y\,\mathrm{Gr}^{1/4}). \tag{2}$$

The function f is determined from the conditions for conservation of the self-similar nature of the solution and the exponential decrease in vorticity on the outer boundary of the boundary layer.

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We put the solution in the boundary layer in the form of asymptotic expansions

$$\Psi(x,\ y;\ \mathrm{Gr}) = \mathrm{Gr}^{-1/4}\Psi_0(X,\ y\,\mathrm{Gr}^{1/4}) + c_1\mathrm{Gr}^{-1/3}\Psi_{0x}(X,\ y\,\mathrm{Gr}^{1/4}) + \\ + \mathrm{Gr}^{-1/2}\Psi_1(X,\ y\,\mathrm{Gr}^{1/4}) + \mathrm{Gr}^{-3/4}\Psi_2(X,\ y\,\mathrm{Gr}^{1/4}) + \dots, \\ \Theta(x,\ y;\ \mathrm{Gr}) = \Theta_0(X,\ y\,\mathrm{Gr}^{1/4}) + \mathrm{Gr}^{-1/4}\Theta_1(X,\ y\,\mathrm{Gr}^{1/4}) + \\ + c_1\mathrm{Gr}^{-1/3}\Theta_{0x}(X,\ y\,\mathrm{Gr}^{1/4}) + \mathrm{Gr}^{-1/2}\Theta_2(X,\ y\,\mathrm{Gr}^{1/4}) + \dots \tag{3}$$

for $\mathrm{Gr} \to \infty$ with X and $y\mathrm{Gr}^{1/4}$ fixed.

The self-similar solution

$$\Psi_0(X,\ y\,\mathrm{Gr}^{1/4}) = X^{3/4}F_0(\eta),\ \ \Theta_0(X,\ y\,\mathrm{Gr}^{1/4}) = H_0(\eta), \\ \Psi_1(X,\ y\,\mathrm{Gr}^{1/4}) = f\Psi_{0x} + F_1(\eta),\ \Theta_1(X,\ y\,\mathrm{Gr}^{1/4}) = f\Theta_{0x} + X^{-3/4}H_1(\eta), \\ \Psi_2(X,\ y\ \ \mathrm{Gr}^{1/4}) = \frac{1}{2}f^2\Psi_{0xx} + fF_{1x} + X^{-3/4}F_2(\eta),\ \Theta_2(X,\ y\,\mathrm{Gr}^{1/4}) = \\ = \frac{1}{2}f^2\Theta_{0xx} + f(X^{-3/4}H_1)_x + X^{-3/2}H_2(\eta),\ \eta = y\,\mathrm{Gr}^{1/4}X^{-1/4}$$

exists, if

$$f(X,\ y\,\mathrm{Gr}^{1/4}) = -y\,\mathrm{Gr}^{1/4} + \frac{4}{3}\alpha X^{1/4};\ \alpha = F_1(\infty)/F_0(\infty), \tag{4}$$

and F_i and H_i are determined from the same set of problems as in [5].

On the leading edge

$$X = \left(-\frac{4}{3}\alpha\right)^{4/3}\mathrm{Gr}^{-1/3},\ \text{i.e.}\quad X \sim \frac{1}{L}\ , \tag{5}$$

and, hence, its relative effect is more significant, the smaller the plate length L.

Expression (4) for the deformation function enables us to find the constant in the characteristic solution

$$c_1 = -\frac{4}{3}\alpha \tag{6}$$

owing to the fact that the characteristic solutions are due to indeterminacy in details of the flow at the leading edge.

Finally, for the local heat-transfer coefficient

$$\mathrm{Nu}_x = -H_0'(0)\mathrm{Gr}_x^{1/4} + \frac{1}{3}\alpha H_0'(0) + \frac{1}{3}\alpha H_0'(0)\mathrm{Gr}_x^{-1/12} - \left[H_2'(0) + \frac{5}{18}\alpha^2 H_0'(0)\right]\mathrm{Gr}_x^{-1/4} + O(\mathrm{Gr}_x^{-1/2}). \tag{7}$$

In particular, for Pr = 0.72,

$$\mathrm{Nu}_x = 0.357\,\mathrm{Gr}_x^{1/4} + 0.179 + 0.179\,\mathrm{Gr}_x^{-1/12} + 1.113\,\mathrm{Gr}_x^{-1/4} + O(\mathrm{Gr}_x^{-1/2}). \tag{8}$$

Before comparing Eqs. (1) and (8) with the experimental data, we turn our attention to the violation of the universality of the relation $\mathrm{Nu}_x = f(\mathrm{Gr}_x)$ when $\mathrm{Gr}_x \le 5 \cdot 10^3$. The application of similarity theory to the analysis of free-convection heat transfer shows that the generalized criterial relation can be written in the form [6]

$$\mathrm{Nu}_x = f(\mathrm{Gr}_x,\ \mathrm{Pr},\ \beta\Delta T). \tag{9}$$

In many cases the effect of the upthrust is characterized sufficiently well by the criterion Gr, and the effect of $\beta\Delta T$ can be ignored. However, at sufficiently small values of Gr the effect of the complex $\beta\Delta T$ cannot be ignored. This is well illustrated in Fig. 1, where the

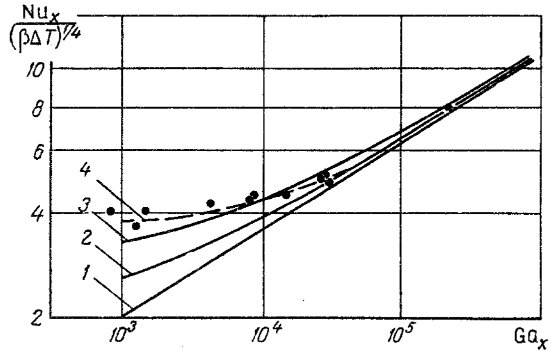

Fig. 1. Comparison of results of calculations and experiments: 1) boundary-layer theory; 2) Eq. (1); 3) Eq. (8); 4) approximation of experimental data of [1].

results of the experiments in [1] are treated in the form of a plot of $Nu_x/(\beta\Delta T)^{1/4}$ against Ga_x. Figure 1 also shows the theoretical curves corresponding to Eqs. (1) and (8) and boundary-layer theory. Inclusion of the effect of the leading edge leads to a more accurate description of the change in the heat-transfer coefficient on reduction of Ga_x. The error when $Ga_x \sim 10^3$ is 20%, whereas the error of boundary-layer theory is 200%. The disagreement between the theoretical and experimental data can be attributed to the discarding of terms of the order $O(Gr_x^{-1/2})$ in Eq. (8), and also to the use of a plate of finite thickness in the experiment.

NOTATION

x, longitudinal coordinate; y, transverse coordinate; T, temperature; ρ, density; L, plate length; g, gravitational acceleration; $\Theta = (T - T_\infty)/(T_w - T_\infty)$, dimensionless temperatures; ψ, stream function; β, bulk expansion coefficient; $\Delta T = T_w - T_\infty$, temperature difference; Pr, Prandtl number, ν/a; Gr, Grashoff number, $g\beta L^3\Delta T/\nu^2$; Ga, Galileo number, gL^3/ν^2; Nu, Nusselt number, $\alpha L/\lambda$. Indices: x, local value; w, wall; ∞, surrounding medium.

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